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FACTORS AFFECTING MEAN POWER RESPONSE TO MULTIPATH RAYS ARRIVING AT DIFFERENT ELEVATION ANGLES

The power response to a source along the main beam, assumed to be in the y-z plane, was given in QPR No. 31, equation (10), page 30, as

$$|A^{2}(\frac{\pi}{2}, e_{s})| > = \sum_{m=1}^{M} |B_{m}|^{2} > N \{1+(N-1)\}$$

 $\mathrm{e}^{-\left(c\frac{2}{y}k^2\left(\sin\theta_{\mathrm{m}}-\sin\theta_{\mathrm{s}}\right)^2+\left(\frac{2}{z}k^2\left(\cos\theta_{\mathrm{m}}-\cos\theta_{\mathrm{s}}\right)^2\right)\right)}$

where it was assumed that elements are distributed normally in dimensions \mathbf{x} , \mathbf{y} , \mathbf{z} , with standard deviation $\mathbf{c}_{\mathbf{x}}$, $\mathbf{c}_{\mathbf{y}}$, and $\mathbf{c}_{\mathbf{z}}$. $\mathbf{\theta}_{\mathbf{m}}$ are co-latitude angles of arrival of ray \mathbf{m} , $\mathbf{m} = 1, 2, \ldots \mathbf{M}$, and ray magnitude is $\mathbf{B}_{\mathbf{m}}$. The exponential factors determine the gain variation with element spread and focusing angle $\mathbf{t}_{\mathbf{s}}$ (measured with respect to the vertical). To better see how $\mathbf{c}_{\mathbf{y}}$ and $\mathbf{c}_{\mathbf{z}}$ affect the beam width in elevation calculations were made of the two factors.

$$I_y(\theta_m, \theta_s) = e^{-c_y^2 k^2 (\sin(\theta_m - \sin(\theta_s))^2)}$$

and

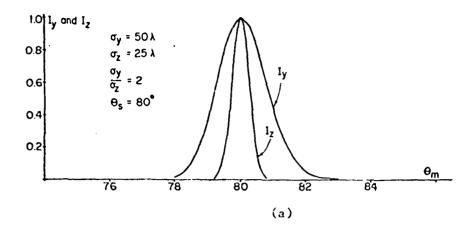
$$I_z(\hat{\epsilon}_m, \hat{\epsilon}_s) = e^{-c_z^2 k^2 (\cos \hat{\epsilon}_m - \cos \hat{\epsilon}_s)^2}$$

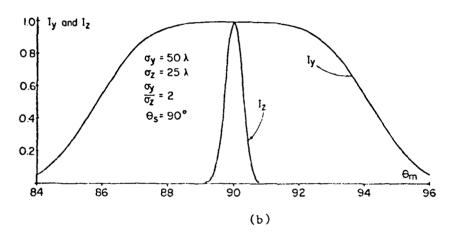
Figure 3.1 (a),(b),(c),(d),(c),(f) shows some representative results for T_y and T_z as a function of θ_m with θ_s a parameter, and Table I shows the half power beamwidth in elevation, denoted θ_m corresponding to each function. Note that when $\theta_s = 90^\circ$, the array is focused along the y-axis and the mean beamwidth is symmetrical around the y-axis in the y-z plane. When θ_s is set to 80° or 85°, T_y is a bimodal function of θ_m with a local minimum at $\theta_n = 90^\circ$. The local minimum is semetimes challow and sometimes quite deep. When T_y falls below 0.5 at $\theta_n = 90^\circ$, T_0 is 1 in the beautifit of each hump; when T_y is above 0.5 at $\theta_n = 90^\circ$, the beautifit given is the total range between half power points and runs from a point below the horizontal. For instance, for $\theta_y = 20^\circ$, $\theta_y = 40^\circ$, $\theta_y = 85^\circ$ the 0.5 power beautiff denoterined by T_y ranges from 81.75° to 98.25°, a range of 16.5°.

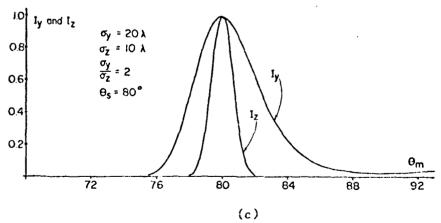
As one right expect by does not esmally strongly influence the beaswidth

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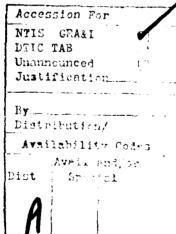
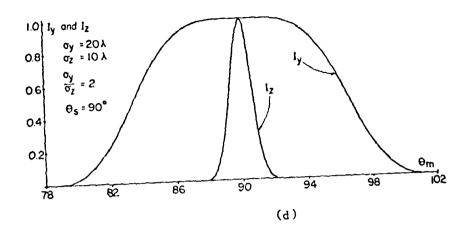
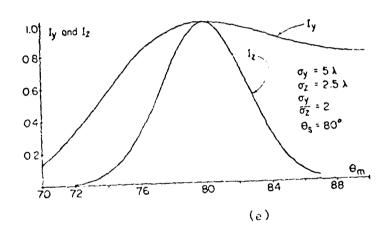


FIGURE 3.1. FACTORS I AND I DETERMINING VERTICAL BEAUWIDTH. (a,b,c)





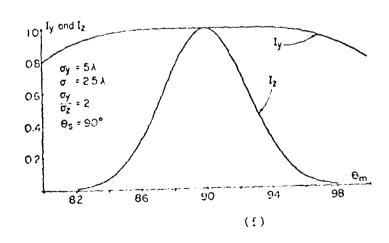


FIGURE 3.1. FACTORS $\mathbf{I}_{\mathbf{y}}$ AND $\mathbf{I}_{\mathbf{z}}$ DETERMINING VERTICAL BEAMWIDTH. (d.,.,f)

(θs	$\frac{\Delta \theta}{m}$ on I_z	$\frac{\Delta \theta_{m}}{m}$ on I_{y}
$\sigma_{\mathbf{y}} = 5\lambda$ $\sigma_{\mathbf{z}} = 2.5\lambda$	\ \ \ 85° \ \ 80°	6.1° 6.1° 6.1°	> 20.0° 28.2° 33.2°
$\sigma_{\mathbf{y}} = 20$ $\sigma_{\mathbf{z}} = 10\lambda$	\begin{cases} 90\\ 85\\ 80\\ \end{cases}	1.6° 1.5° 1.5°	13.2° 16.6° 4.5°
$\sigma_{\mathbf{y}} = 50$ $\sigma_{\mathbf{z}} = 25$	\begin{cases} 90° \ 85° \ 80° \end{cases}	0.6° 0.6° 0.6°	8.4° 3.75° 1.75°
$ \sigma_{\mathbf{y}} = 5\lambda \\ \sigma_{\mathbf{z}} = \lambda $	\begin{cases} 90° \ 85° \ 80° \end{cases}	15.2° 15.3° 14.2°	> 20.0° > 20.0° > 20.0°
$\sigma_{\mathbf{y}} = 20\lambda$ $\sigma_{\mathbf{z}} = 4\lambda$	\begin{cases} 90° \ 85° \ 80	3.8° 3.8° 3.8°	13.0° 16.5° 4.5°
$\sigma_{\mathbf{y}} = 50\lambda$ $\sigma_{\mathbf{z}} = 10\lambda$	\begin{cases} 90\circ \ 85\circ \ 80\circ \end{cases}	1.5° 1.5° 1.5°	8.4° 3.75° 1.75°

TABLE I MEAN 1/2-POWER BEAMWILTH, $\Delta\theta_{m}$, WITH NORMALLY DISTRIBUTED ELEMENT POSITIONS.

in elevation, though for $\sigma_y = 50\lambda$, $\sigma_z = 10\lambda$, and $\theta_z = 80^\circ$ the two functions are nearly equal. As a rule with $\sigma_y/\sigma_z = 2$, I_z established the pattern in elevation and it is sufficient to consider it and to let $I_y = 1$.

In the work discussed below on the noise correlation we consider elements uniformly dispersed in depth where here we have assumed normally distributed elements. Extraordinary differences between these two cases are not expected though. Similar computations of the uniformly distributed case were therefore not carried out.

Paul Yeh Fred Haber

MULTIBEAM NOISE CORRELATION

Consider the system in Figure 3.2, QFR No. 31. The branch signal level, when the array is focused in the vertical plane containing the Y-axis and at an

angle θ_{si} to the vertical, is given by (2) of QPR 31. The equation is rewritten here

$$A(\theta_{si}) = \sum_{m=1}^{M} \sum_{n=1}^{K} \beta_m e^{j\{ky_n(\sin\theta_m - \sin\theta_{si}) + kz_n(\cos\theta_m - \cos\theta_{si}) + \phi_m\}}$$
(1)

The complex weight

$$W_{i} = A_{i} e^{-j\psi_{i} \operatorname{sgnf}}$$
 (2)

shown in the figure referred to is given by

$$W_{i} = A^{h}(\rho_{si}) \tag{3}$$

when noise levels into all the branches have equal mean square value and are independent from branch to branch. The total system signal output is, in this case,

$$W = \frac{7}{1} A(^{\circ}_{si}) W_{i} = \frac{7}{1} A^{2} (^{\circ}_{si}) = \frac{5}{1} A_{i}^{2}$$
(4)

and the total noise outut has a mean square value given by the second expression p. 35 of OPR 31. namely.

$$\langle N^2(t) \rangle = \frac{N}{\sum_{n=1}^{N}} \langle n_n^2(t) \rangle = \frac{1}{\sum_{i=1}^{N}} \frac{1}{\sum_{j=1}^{N}} \langle A_j A_j \cos(t_{n,i} - \phi_{n,j} + \psi_j - \psi_j) \rangle$$
 (5)

where the A_i are defined by (2). The angles in (5) are given by the following

$$\phi_{ni} = -k[y \sin \theta_{si} + z \cos \theta_{si}]$$

$$M N$$
(6)

$$\psi_{i} = \tan^{-1} \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin[x_{n} + x_{m}]}{\sin[x_{n} + x_{m}]}}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin[x_{n} + x_{m}]}{\sin[x_{n} + x_{m}]}}$$
(7)

where we have used (I) and (6) in writing (7) and where we have denoted

$$\frac{1}{nr_0} = k \left(y_n \sin \frac{1}{r} + z_n \cos z_m \right) \tag{8}$$

If the noise processes in the different branches were uncorrelated the output mean square noise level would be

$$\langle N^{2}(t) \rangle = \sum_{n=1}^{N} \langle n_{i1}^{2}(t) \rangle \sum_{i=1}^{N} \langle A_{i}^{2} \rangle$$
 (9)

The output signal-to-noise ratio (SNR) is here defined by

$$\gamma = \frac{\langle w^2 \rangle}{2 \langle N^2(t) \rangle} = \frac{\left(\sum_{i=1}^{I} A_i^2\right)^2}{2N \langle n_n^2(t) \rangle \sum_{i=1}^{I} \langle A_i^2 \rangle}$$
(10)

where we have assumed mean noise power at all elements to be equal and where we have used (2), (3), (4) and (9) to write (10). If the A_i were not random variables (or if they were to have small variance), γ would be the sum of SNR's of the branches, a result well known for maximal ratio combining of diversity branches in communications. In our situation the branch noises may not be uncorrelated and there is the possibility of larger noise levels. We examine conditions which will give uncorrelated noises. (5) can be expended to give

 A_i and ψ_i are the amplitude and phase of the complex amplitude on branch i as given by (1). These random variables are not determined by the position random variables y_i and z_i because of the presence of ϕ_i in the exponent of (1) which is uniformly distributed in (0, 2%). On the other hand, the ϕ_{ni} are determined only by the position random variables y_i and z_i . Thus the pair (A_i, ψ_i) are independent of ϕ_{ni} for all i. Furthermore from (6) we see that ϕ_{ni} is a linear function of the random variables y_i and z_i . We assume the latter to be symmetrically distributed around zero so that $< \sin(\phi_{ni} - \phi_{nj}) > = 0$. (11) thus reduces to

$$N^{2}(t) > = \sum_{n=1}^{N} \langle n_{n}^{2}(t) \rangle \sum_{i=1}^{I} \sum_{j=1}^{I} \langle A_{i}A_{j} \cos(v_{i} - v_{j}) \rangle \langle \cos(\phi_{ni} - \phi_{nj}) \rangle (12)$$

We point out that if either $<\cos(\psi_{ni}-\epsilon_{nj})>$ or $<\mathbf{A_iA_i}\cos(\psi_i-\psi_j)>$ is zero for all $i\neq j$ (12) reduces to (9)—that is, the uncorrelated branch noise case is obtained. We investigate

$$< \cos(\phi_{ni} - \phi_{nj}) > = < \cos k[y_n(\sin \theta_{si} - \sin \phi_{sj}) + z_n(\cos \theta_{si} - \cos \theta_{sj})] > (13)$$

since it is much less complicated than $<\cos(z_i-z_i)>$. The angles θ_{si} of interest to us are in the range of about 50° to 100°. It can be demonstrated that $(\sin\theta_{si}-\sin\theta_{sj})<<(\cos\theta_{si}-\cos\theta_{sj})$ for angles in this range. Also, if we are to have high vertical resolution, the variance of z_n will have to be of the same order as the variance of y_n . Thus we can argue that (13) can be approximated by

$$\langle \cos(z_{ni} - z_{nj}) \rangle \langle \cos k z_{n} \langle \cos z_{si} - \cos z_{sj} \rangle$$
 (14)

so that (12) becomes

$$< N^{2}(t) > = \sum_{M=1}^{N} < n_{n}^{2} > \sum_{i=1}^{I} \sum_{j=1}^{I} < A_{j}A_{j} \cos(U_{i}-U_{j}) >$$
 (15)

•
$$< \cos kz_n(\cos t_{si} - \cos \phi_{sj}) >$$

If the rv z_n is here assumed uniformly distributed in a range (-h,h) then the rv $(:_{ni}-:_{nj})$ will be correspondingly uniformly distributed in a range $(-a_{ij}, a_{ij})$. We therefore have

$$<\cos(\frac{1}{n_i} - \frac{1}{n_j})^{\frac{1}{2}} = \frac{\sin a_{ij}}{a_{ij}}$$
(16)

where

$$a_{ij} = kh(\cos z_{si} - \cos z_{sj})$$
 (17)

(16) suggests that to make $< \cos(\phi_{ni} - \phi_{nj}) > = 0$ for $i \neq j$, branch focii be placed so that $a_{ij} = m\dot{\pi}$, m an integer.

The $\theta_{\rm si}$ will all be concentrated around 90°, generally in the range (80°, 190°). Thus we are led to write

$$\hat{\theta}_{si} = \theta_{si} - 90^{\circ}$$

so that

$$\cos \theta_{si} = -\sin \hat{\theta}_{si} = -\hat{\theta}_{si}$$

and using (17)

$$a_{ij} = kh \left(\theta_{sj} - \theta_{si}\right) \tag{18}$$

The angular separation between beams to get uncorrelated noise outputs at the different branches should therefore be

$$\theta_{sj} - \theta_{si} = \frac{m\pi}{kh}$$

If, for instance, $h = 25\lambda$ (meaning that the array size in depth is 50 wavelengths which at Hz implies a depth of about 750 meters) adjacent beams ought to be spaced

$$\Delta\theta_{s} = \frac{\pi}{kh} = 1/50 \text{ rad} = 1.15^{\circ}$$

in order for the roise variables entering into the final summer to be uncorrelated.

The next step to be taken will be to determine the array power response with array output processed as described above. If the multipath rays were to come in on the center lines of the vertically spread beams the overall array response would be maximized. Some beams are expected however to see no incoming rays, others may see more than one ray. In the latter case the multipath is not resolved by the array processor and the combined rays add non-coherently. The corresponding diversity branch will see a fluctuating level depending on the relative ray phases. While the processor cannot improve the signal level in this case the weighting circuit will take account of this fluctuation to maximize the signal to noise ratio by suppressing the branch output if the signal component is small and amplifying the branch output if the signal component is large.

The calculation of these effects is time consuming. We plan to carry out a simulation of this next step by assuming a fixed number of rays arriving, each uniformly distributed over a range of latitude angles about ±10% relative to the horizontal. Independent noise at each sensor will be assumed. Beams will be spaced as specified above covering the same range of latitudes and the statistical properties of the array output SNR will be found. This calculation will also serve as a test of assumptions made earlier to simplify calculations of noise output.

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   24 - APPROACH: (U) INVESTIGATE PHASE DECORRELATION EFFECT ON ARRAY GAIN
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      OF A PHINDOM APPAR THEORY HAS BEEN DEVELOPED. VALLEY FORGE RESEARCH
      CENTER GUARTERLY PROGRESS REPORT, FEB. 1979 (U)
    13 - WORK UNIT START DATE: MAR 77
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